Lessons Learned in Retention Analysis: Recipes for Success

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Abstract

Pacific Gas and Electric (PG&E) and Quantum Consulting (QC) recently completed a retention study on the most significant measures installed in PG&E's 1994 and 1995 Commercial Energy Efficiency Incentives programs. A significant data collection effort has been allocated to this study. In 1995 and 1996 a total of 600 participants were on-site audited in order to build a retention panel of installed equipment. This panel includes information on thousands of T-8 and electronic ballasts, for example. In 1997 and 1998, the majority of these sites were revisited to determine which equipment had been removed or had failed. The timing of the removal/failure, whether the failed equipment was warrantied, and what the equipment was replaced with, was also determined. PG&E and QC conducted the analysis of this data using a number of statistical survival analysis techniques to estimate the equipment's effective useful life.

Because retention studies are now being planned by many utilities, this paper will be of interest to evaluators across the country for a number of reasons. First we present a number of methodologies used to conduct survival analysis on commercial lighting and HVAC measures. Secondly, the results of this paper will be extremely useful, as it provide estimates of the effective useful life for the most commonly installed commercial technologies. Further, we identify the distribution of the equipment's life cycle. We also present all of the issues that we have encountered over the course of the analysis, and provide recommendations on how to address them.

Introduction

Pacific Gas and Electric Company (PG&E) has been running commercial energy efficiency incentives programs for over a decade now, as have many utilities across the country. One component of program design that has always been paramount to the program's success is its cost-effectiveness. The life cycle of the program measures is a key element in determining cost-effectiveness. Perhaps the most commonly installed commercial measure is the T-8 lamp with electronic ballast. Prior to 1990, this measure was considered to be somewhat unreliable. During the early 1990's, the measure's reliability markedly increased, although its life cycle was still uncertain. Now that many of these more reliable T-8s have been installed for five or more years, many utilities are beginning to conduct retention studies, with the objective being to re-evaluate the measure's life cycle.

PG&E and Quantum Consulting (QC) recently completed a retention study on the most significant measures installed in PG&E's 1994 and 1995 Commercial Energy Efficiency Incentives (CEEI) programs. A significant data collection effort has been allocated to this study. In 1995 and 1996 a total of 600 participants were on-site audited in order to build a retention panel of installed equipment. This panel includes information on tens of thousands of T-8 and electronic ballasts, for example. In 1997 and 1998, the majority of these sites were revisited to determine which equipment had been removed or had failed. The timing of the removal/failure, whether the failed equipment was warrantied, and what the equipment was replaced with, was also determined. PG&E and QC have analyzed this data using a number of statistical survival analysis techniques to estimate the equipment's effective useful life (EUL). PG&E defines the EUL as follows:

Effective Useful Life (EUL) – An estimate of the median number of years that the measures installed under the program are still in place and operable.

This retention study was focused on the seven measures that provided the majority of energy savings in PG&E's CEEI programs. These measures include the following:

- T-8 Lamps and Electronic Ballasts
- Optical Reflectors with Flourescent Delamping
- High Intensity Discharge (HID) Lamps for Indoor Applications
- Adjustable Speed Drives (ASD) HVAC Fans, 50 HP Maximum
- Water Chillers >= 300 Tons
- Cooling Towers
- Energy Management Systems (EMS) for HVAC

This paper presents the results of the retention study conducted on PG&E's 1994 and 1995 CEEI programs.

Methodology

The purpose of this retention study was to collect data on the fraction of installed measures in place and operable in order to produce a revised estimate of its EUL. The ultimate goal is to estimate the EUL (or the median number of years that the measure is still in place and operable), which can be realized by identifying the measure's survival function. For this study, the survival function describes the percentage of measures installed that are still operable and in place at a given time. At any given time, the hazard rate is the rate at which measures fail or are removed. Survival analysis is the process of analyzing empirical failure/removal data in order to model a measure's survival function. As much as possible, we have attempted to employ classical survival analysis techniques to our study approach.

Our overall approach was to apply survival analysis to our collected retention data in order to develop a survival function for each of the studied measures. Some of the common survival functions take on the log-logistic, exponential, Weibull, lognormal, and gamma distributions. For this retention study, we have examined each of these distributions. We have used the SAS System and the SAS companion guide, "Survival Analysis Using the SAS System[1]," in order to estimate the survival functions based on the retention data for each of our studied measures.

An important issue to keep in mind for this analysis is the definition of survival. Recall that the EUL is defined as the median number of years that the measures installed under the program are still in place and operable. Therefore, to "survive", a measure must not have been removed or have failed.

[1] Allison, Paul D. 1995, "Survival Analysis Using the SAS System, A Practical Guide", SAS Institute, NC.

Unfortunately, it is likely that the underlying distribution of measures having failed is very different than the distribution of removals.

There is much literature to suggest, for example, that electronic ballast failures follow an exponential distribution. The exponential survival function has a constant hazard rate. In other words, the rate at which electronic ballasts fail is constant over time. This belief is founded on the fact that electronic devices are likely to fail at any point in time with equal probability. Because electronic ballasts may have anywhere from 30 to 120 parts, plus more than twice as many solder joints as there are parts, it is likely that the ballast may also fail at any point in time, with equal probability.[2]

However, the removal of an electronic ballast is more dependent on human interaction. For example, consider the act of remodeling, or upgrading the system as new technologies emerge. Both of these actions are likely to occur in the latter stage of the equipment's life. However, if the customer is not satisfied with the technology, the removal may occur early on in the equipment's life. Whatever the case may be, it is likely that the survival function of equipment removal differs from the survival function of the equipment failure.

For this study, the vast majority of measures were in place less than five years (few were installed prior to 1994, and follow-up data collection was conducted no later than the end of 1998). Because the ex ante EUL is 15-20 years for most measures, it was unlikely from the start that our data would be capable of accurately estimating this mixture probability density function of failures and removals.

Our overall approach consists of four analysis steps that were used to estimate each of the studied measures' EULs:

1. ***Compile summary statistics*** on the raw retention data. For some measures, it was sufficient to only look at the raw data, because for some measures, all of the sampled equipment was still in place and operable.
2. ***Visually inspect*** the retention data. By calculating the cumulative percentage of equipment that had failed in a given month, and plotting this percentage over time, an empirical survival function emerges.
3. ***Develop a trend line*** from the survival plots. Using the plots developed in (2) above, we estimated a trend line using standard linear regression techniques. We attempted to model the trend as a linear and an exponential function. In each case, we plotted the resulting trend line and visually compared it to the survival plot developed in (2). Furthermore, we used the resulting trend line to estimate the EUL.
4. ***Develop a survival function*** using classical survival techniques. Using the SAS System and the SAS companion guide, "Survival Analysis Using the SAS System," we modeled the survival function assuming five of the most common survival distributions: exponential, log-logistic, lognormal, Weibull and gamma. In each case, we plotted the resulting distribution and visually compared it to the survival plot developed in (2). Furthermore, we used the resulting survival function to estimate the EUL.

Sample Design

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[2] Energy User News, Vol. 23 No. 10, October 1998. Electronics, Energy Products and Life-Cycle Costing: 28.

It is important to note that the unit of analysis for a retention study is not a site, but a unit of measure. For example, for lighting measures, the unit of analysis is generally a ballast. For chillers, the unit of analysis is tons. Therefore, a single site may consist of hundreds, or even thousands of units. In this case, each sample unit is not independent of the others. This is important to consider when developing confidence intervals, as will be discussed in more detail below.

Table 1 below provides the sample frame that was available for analysis for this study. Shown are the number of sites surveyed, and the number of units installed across all sites. For all measures, a census was conducted on the available panel data.

Table 1. Final Sample Disposition

End Use	Technology	Number of Sites Contacted	Units	Total Number of Units
Lighting	Optical Reflectors w/ Fluor. Delamp	51	Lamps	4,883
	T8 Lamps and Electronic Ballasts	138	Ballasts	12,085
	High Intensity Discharge	47	Fixtures	946
HVAC	ASD	16	HP	548
	Chiller	7	Tons	4,834
	Cooling Tower	24	Tons	10,022
	EMS	21	Systems	21

Data Collection Strategy

The data collection effort surrounding the survival analysis included a combination of telephone and on-site surveys. When possible, these data were gathered using telephone surveys, with alternate data collection using on-site audits where installations were too complex to be supported by self-reported data. Roughly half of the survival analysis surveys were conducted over the telephone, with the other half requiring an on-site visit. In general, on-sites were required for many of the lighting end use installations, while HVAC equipment survival was more readily verified using the telephone interview only. The following outlines the data collection procedures:

For each unit of equipment in the retention panel, it was determined whether (1) the equipment was still installed, and (2) if it was operable. If the equipment was not in place or was not operable, it was determined when it was removed or stopped operating according to the owner or operators best recollection. Reasons for removal or failure to operate were also collected. If equipment as replaced, it was determined if the equipment was replaced with a standard, equivalent or higher efficiency technology. Finally, it was determined if replaced equipment was done so under warranty.

Results

This section presents the final results of the 1994 CEEI Retention Study.

Compile Summary Statistics

For some measures, it was sufficient to only look at the raw data, because for some measures, all of the sampled equipment was still in place and operable. For measures that did exhibit some failures and removals, it was clear that such a small percentage of failures and removals had occurred, that it would be nearly impossible to model the equipment's survival function. Table 2 presents the percentage of measures that were found to have failed or been removed over the study period.

Table 2 clearly demonstrates that for the Chiller and Cooling Tower measures, it will be impossible to develop a survival function or an ex post EUL estimate, since no failures or removals occurred during the study period. Furthermore, the Delamp and EMS measures exhibited only one or two failures or removals in the sample. With such limited data on failures, a reliable survival function cannot be developed nor can an ex post EUL estimate. Because of this, no further analysis was conducted on the Chiller, Cooling Tower, Delamp, or EMS measures.

Table 2. Summary Statistics on Raw Retention Data

End Use	Technology	Number of Sites Contacted	Units	Total Number of Units	Number of Units in Place and Operable	Percent Failed, Removed, Replaced
Lighting	Optical Reflectors w/ Fluor. Delamp	51	Lamps	4,883	4,881	0.04%
	T8 Lamps and Electronic Ballasts	138	Ballasts	12,085	11,834	2.08%
	High Intensity Discharge	47	Fixtures	946	895	5.39%
HVAC	ASD	16	HP	548	533	2.74%
	Chiller	7	Tons	4,834	4,834	0.00%
	Cooling Tower	24	Tons	10,022	10,022	0.00%
	EMS	21	systems	21	20	4.76%

Visual Inspection

Using the raw retention data, we developed empirical distributions of the survival function for each of the studied measures. This step clearly illustrated that for each studied measure, there was not enough data over time to support an accurate estimate of the survival function. For this study, the vast majority of measures were in place less than five years (few were installed prior to 1994, and follow-up data collection was conducted no later than the end of 1998). Because the ex ante EUL is 15-20 years for most measures, our data were not capable of accurately estimating the survival function of failures and removals.

Figure 1 provides the empirical survival function for the three studied measures that had a sufficient number of failures occur during the study period to produce any survival function.

In order to develop the empirical survival functions, some assumptions were necessary. For example, when failures or removals were reported during the customer surveys, failure and removal dates were not always provided. Also, most of the measures were still in operation at the time of survey. Therefore, there was no failure time to record. These two issues are referred to as interval and left-hand censoring. Where there are approaches for dealing with these issues using classical survival analysis, there are no stead-fast rules for simple empirical analyses, such as this. These assumptions are discussed in more detail in Issues Section , below. It should be noted, however, that the empirical survival functions primary purpose is to allow the analyst to obtain an easy to develop picture of how the data are behaving, and get an early indication of what the survival function looks like.

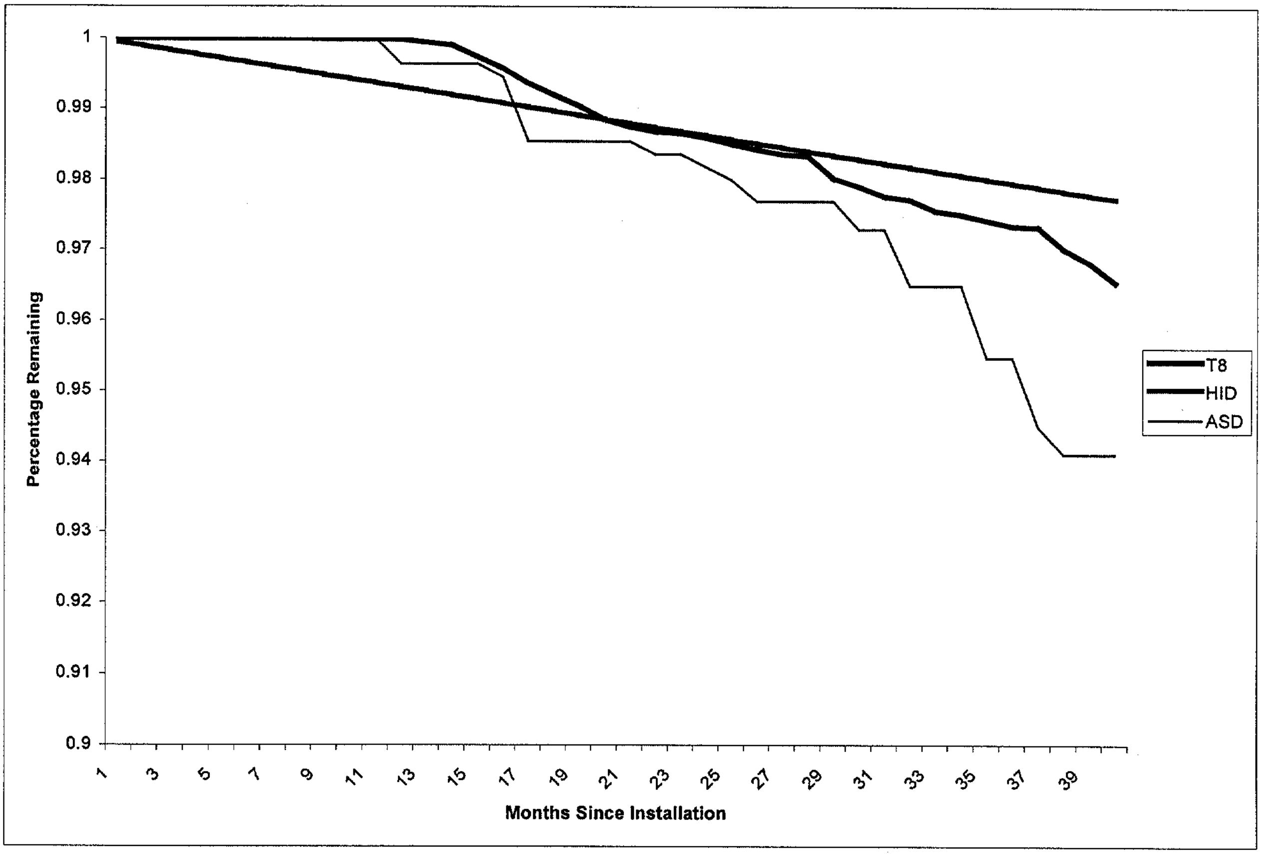

Figure 1. Empirical Survival Functions

Develop a Trend Line

Using the empirical functions developed above, a trend line was estimated using standard linear regression techniques. We modeled the trend as a linear and an exponential function (by taking the log of the percentage operable). In each case, we plotted the resulting trend line and visually compared it to the empirical survival function developed above. Figure 2 presents a comparison of the linear and exponential trend lines and the empirical survival function for the T8 measure.

The results of the trendline regressions are provided in Table 3 for each of the four measures. Also provided in Table 3 is the estimated EUL for each measure. Clearly, the results of the linear and exponential trendline estimate indicate that the ex post EUL estimates are significantly larger than the *a priori* estimates, which are all 16 years.

For a linear survival function, the EUL (median life) is calculated as:

EUL = (0.5 – intercept)/slope

For an exponential survival function, the EUL (median life) is calculated as:

$$EUL = \ln(2)/slope$$

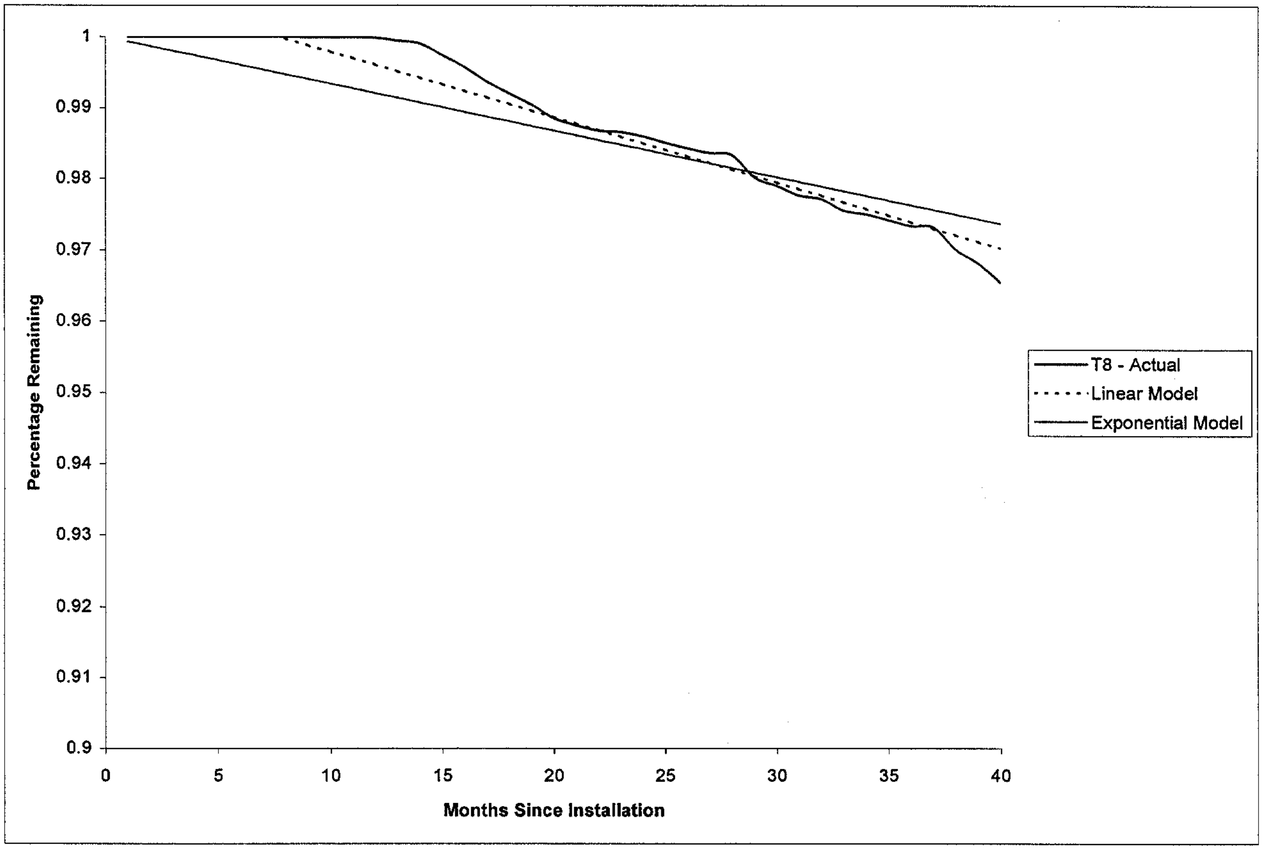

Figure 2. Comparison of Linear and Exponential Models to Empirical Function for T8 Measure

Table 3. Regression Results of Linear and Exponential Trendlines and Resulting EUL Estimates

Measure	Intercept	t-Statistic	Slope	t-Statistic	EUL
Linear Model					
T8 Lamp and Electronic Ballast	1.01	1,193	-0.0009	-25.67	46
Interior HID	1.01	279	-0.0013	-8.39	33
Adjustable Speed Drive	1.01	490	-0.0015	-17.16	28
Exponential Model					
T8 Lamp and Electronic Ballast	-	-	0.0007	22.33	87
Interior HID	-	-	0.0008	8.26	76
Adjustable Speed Drive	-	-	0.0011	17.27	54

Again, because the empirical functions are based on a number of assumptions, the trendlines developed based on these functions should be treated with less reliability. It is interested to note, however, that the results of the exponential trendlines developed here are not statistically significantly different than those developed using survival analysis, as discussed below. In fact, for the T8 measure, the results were within a few percent of each other.

Develop a Survival Function

Using classical survival techniques, we modeled the survival function assuming five of the most common survival distributions: exponential, log-logistic, lognormal, Weibull and gamma. In each case, we plotted the resulting distribution and visually compared it to the survival plot developed above. Furthermore, we used the resulting survival function to estimate the EUL. For the T8 and HID measures, annual operating hours was used as a model covariate, as it was assumed that businesses with greater hours of operation would have a higher failure rate. This analysis was conducting in SAS using the LIFEREG procedure. Figure 3 presents the estimated survival functions for each of the five distributions for the T8 measure.

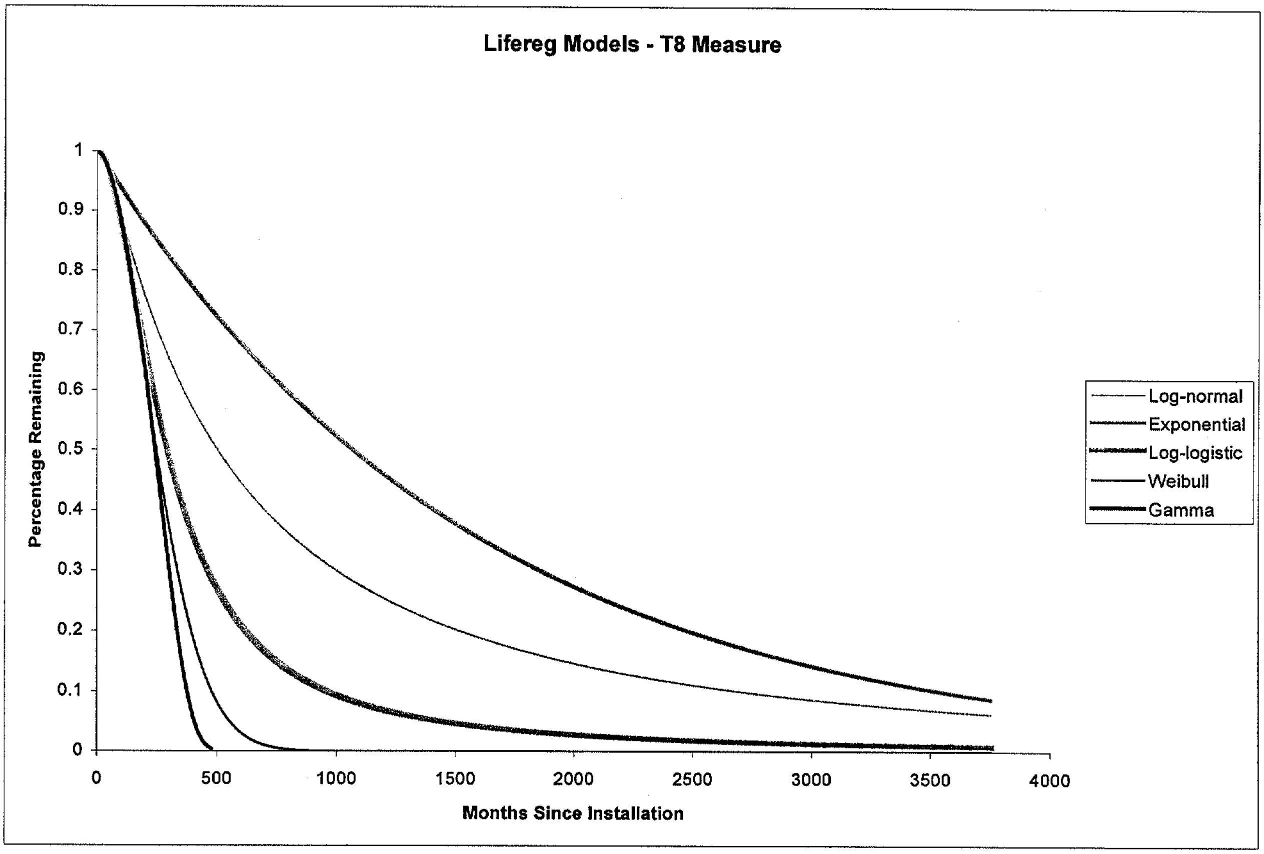

Figure 3. Estimated Survival Functions for the T8 Measure

Table 4 provides the results of the classical survival analysis. Shown are the model results for each measure, and for each type of distribution modeled. Furthermore, the resulting EUL estimates are provided.

Table 4. Comparison of Survival Model Results

Measure	Model		Variable: Intercept	Scale	Ophours	Resulting EUL
T8	Exponential	Parameter Estimate	8.56	1.00	-0.00033	**89.1**
		Standard Error	3.60	0.00	0.00085	31.23
	Logistic	Parameter Estimate	6.36	0.54	-0.00019	**24.3**
		Standard Error	2.37	0.18	0.00053	15.79
	Log-Normal	Parameter Estimate	6.70	1.31	-0.00013	**41.8**
		Standard Error	2.17	0.42	0.00045	34.81
	Weibull	Parameter Estimate	6.42	0.55	-0.00019	**20.6**
		Standard Error	2.41	0.19	0.00053	12.25
	Gamma	Estimate	6.52	0.32	-0.00021	**20.2**
		Standard Error	2.40	0.10	0.00057	7.65
HID	Exponential	Parameter Estimate	14.17	1.00	-0.00177	**100.1**
		Standard Error	10.27	0.00	0.00240	47.24
	Logistic	Parameter Estimate	6.57	0.29	-0.00045	**10.7**
		Standard Error	3.98	0.13	0.00091	3.77
	Log-Normal	Parameter Estimate	6.08	0.63	-0.00030	**11.9**
		Standard Error	2.99	0.26	0.00066	5.23
	Weibull	Parameter Estimate	6.63	0.29	-0.00046	**9.9**
		Standard Error	4.02	0.13	0.00091	3.17
	Gamma	Estimate	7.25	0.10	-0.00058	**10.9**
		Standard Error	4.22	0.06	0.00098	5.72
ASD	Exponential	Parameter Estimate	7.21	1.00	-	**78.1**
		Standard Error	1.12	0.00	-	87.42
	Logistic	Parameter Estimate	5.21	0.44	-	**15.3**
		Standard Error	2.12	0.59	-	32.35
	Log-Normal	Parameter Estimate	5.90	1.18	-	**30.4**
		Standard Error	2.85	1.44	-	86.65
	Weibull	Parameter Estimate	5.22	0.44	-	**13.2**
		Standard Error	2.14	0.59	-	25.38
	Gamma	Estimate	5.71	0.17	-	**18.3**
		Standard Error	0.67	0.20	-	12.25

Final Results

Table 5 summarizes the estimated EULs for each studied measure for each approach and corresponding model. The median EULs are provided, along with the upper and lower confidence bounds, based on the 80 percent confidence interval. For the Delamp, Chiller, Cooling Tower and EMS measures, there was not a sufficient number of failures or removals to estimate an EUL.

Table 5. Summary of Results

		Measures								
		T8			HID			ADS		
Approach	Model	Lower Bound	Median	Upper Bound	Lower Bound	Median	Upper Bound	Lower Bound	Median	Upper Bound
Trendlines	Linear	44	46	48	28	33	38	26	28	30
	Exponential	82	87	92	64	76	87	50	54	58
LIFEREG	Exponential	49	89	129	40	100	161	-34	78	190
	Log-Logistic	4	24	44	6	11	15	-26	15	57
	Log-Normal	-3	42	86	5	12	19	-81	30	141
	Weibull	5	21	36	6	10	14	-19	13	46
	Gamma	10	20	30	4	11	18	3	18	34

Before recommending a methodology to estimate the EUL, it is first important to consider the definition of a confidence interval. Most people mistakenly interpret an 80 percent confidence interval, for example, to mean that there is an 80 percent probability that the true median EUL is contained within the interval provided. This is ***not*** true. The correct interpretation of an 80 percent confidence interval is that if a given experiment is repeated a large enough number of times (say 30 or more), the median obtained from the same model will be contained in the confidence interval 80 percent of the time.

Take for example the exponential distribution modeled for the T8 measure, using the LIFEREG procedure. If we were to repeat our experiment and create a retention panel of 138 sites with 12,085 units originally installed (as was done for this study), there would be an 80 percent probability that the resulting median EUL using the exponential LIFEREG model would result in a value between 49 and 129 years.

Therefore, the results presented above should not be interpreted as data intervals which have an 80 percent probability of containing the true median EUL. One common use of confidence intervals is to identify models that provide results that are not statistically significantly different than zero. As we can see above, many of our model results are not statistically significantly different than zero when measured at the 80 percent confidence level. In fact, the only model from the LIFEREG procedure that produces a statistically significant result for all measures is the gamma distribution.

We point this all out, because based on our extensive analysis of the retention data, we believe that there is insufficient data to provide reliable model results. There may be sufficient sample sizes to produce statistically significant results, but there clearly is not enough data over time to reliably estimate the median EUL. Ideally, the period of data collection will span the expected median life of the measure. This can be illustrated by the sensitivity in the model results.

Take, for example, the five model results based on the LIFEREG procedure for the T8 measure. The median EUL based on the exponential distribution was 89 years, versus only 20 years using the gamma distribution. If we had a sufficient amount of data over time, such that the retention data actually covered the true median, we would expect the median result for the two models to be extremely close! Keep in mind that only about 40 months of valid data were collected for this measure, and that the *a priori* EUL was 192 months. After 40 months, the gamma distribution actually estimated fewer failure/removals than the exponential distribution, as shown below in Figure 4.

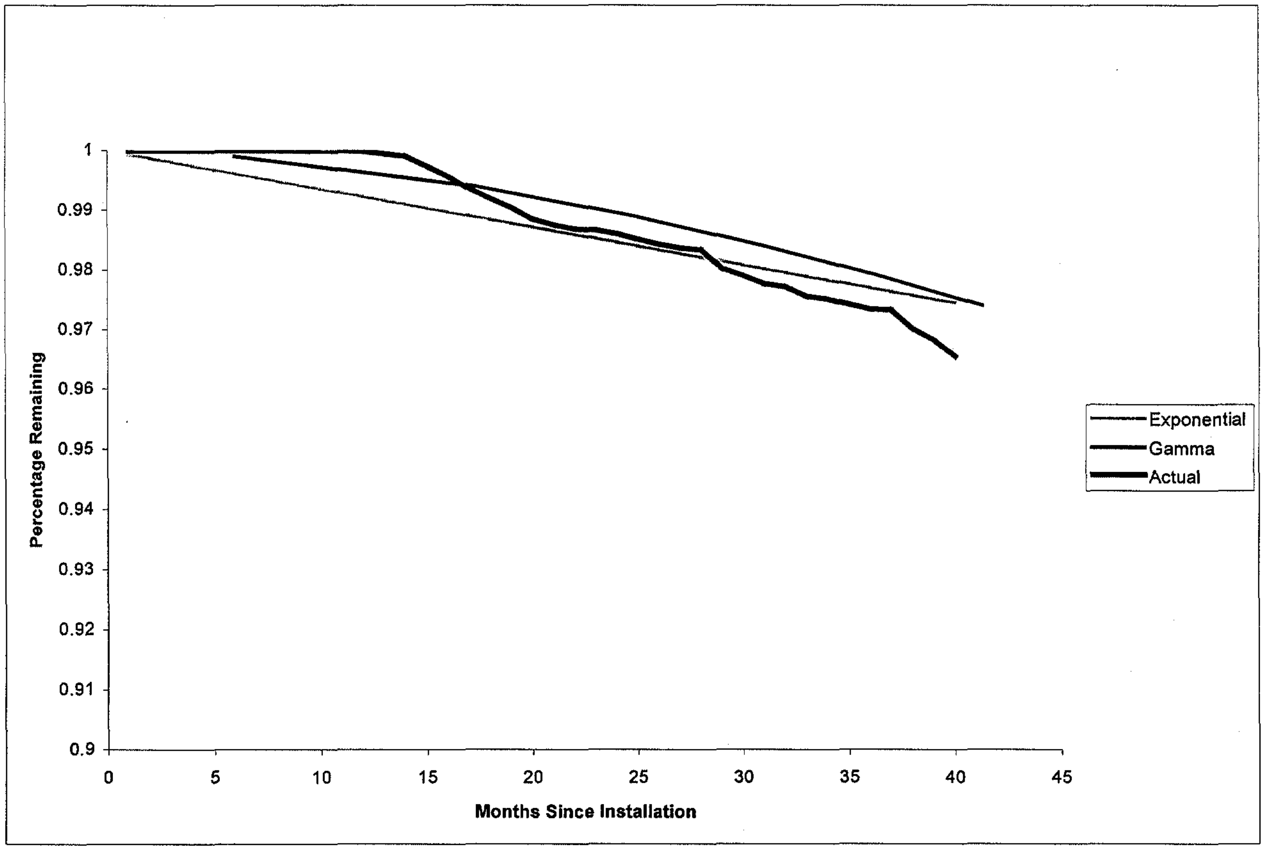

Figure 4. Comparison of Survival Functions for the T8 Measure: Exponential and Gamma versus Empirical Function

Figure 4 further illustrates how close the two models estimate the empirical survival function, and how close the two models are to each other. Beyond the 40 months, however, there is little data for the model to structure the remaining survival function. Consider what happens over the next 200 months, up to the 20th year. As shown in Figure 5, in year 20, the gamma model has reached its median point; whereas the exponential distribution still predicts that 85 percent of the measures are in place and operable. Which model result is better?

Clearly at this point in the measure's life it is not possible to state with much certainty, which model result is superior to the other.

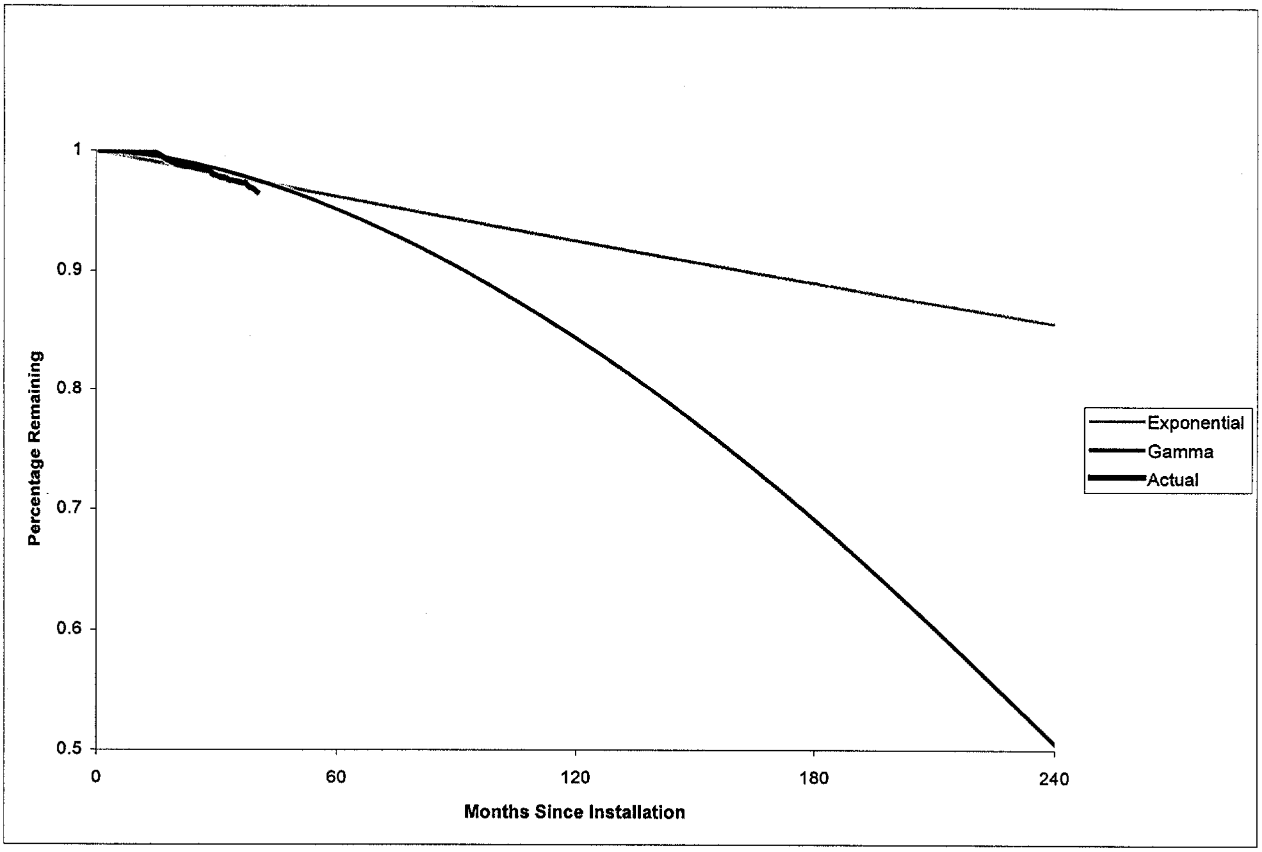

Figure 5. Comparison of Survival Functions Over 20 Years for the T8 Measure: Exponential and Gamma versus Empirical Function

Our recommendation would be to discard all of the model results on the basis that there is insufficient data over the life of the measures. We want to stress that we believe the sample sizes are sufficient. It is only that we have not observed the sample over a long enough period of time. However, if we were to select a result, the following would be our recommendation.

For the three measures that had sufficient failures and removals, all approaches discussed above were implemented. The results based on the summary statistics are not recommended, as they based solely on the overall failure/removal rate observed during the study period. In addition, the results based on the trendlines are not recommended, as they are based on a number of assumptions, as discussed earlier.

Therefore, the recommended results are based on the classical survival analysis using the LIFEREG procedure. Of the five distributions modeled, the gamma distribution is the most adaptive. The LIFEREG procedure models the generalized gamma distribution, which has three parameters. Because this model has at least one more parameter than any of the other distributions, it can take on a wide variety of shapes. In addition, the exponential, Weibull and log-normal distributions are all special cases of the generalized gamma model. But the generalized gamma model can also take on shapes that are unlike any of these special cases. Most importantly, it can have hazard functions with U or bathtub shapes, in which the failure rate (or hazard function) declines, reaches a minimum, and then increases.

Intuitively, then, one would expect the gamma results to provide a better model fit than either the exponential, Weibull or log-normal models (since these are all special cases of the gamma model). As expected, the gamma distribution generally provided the best model fit, as measured by the log-likelihood estimate provided by the LIFEREG procedure. Furthermore, the gamma model is the distribution that provided a result for each measure that was statistically significantly different than zero, measured at the 80% confidence interval[3]. For these reasons, we recommend that the survival function be based on the gamma distribution.

Issues

Missing Failure Dates

One of the most common data collection issues that we encountered was that of missing failure dates. Many customers were able to tell us that equipment had failed, or been removed, but were unable to give us an accurate estimate of the dates surrounding these events. Two common terms used in classical survival analysis are "left-hand censoring" and "right-hand censoring". Left-hand censoring means that it is known that a failure/removal has occurred, but it is unknown when the failure/removal occurred. It is only known that the failure/removal occurred before a certain date.

Right-hand censoring is more common in our data. Right-hand censoring means that at the last time the customer was surveyed, a failure/removal had not occurred, so the time when the equipment will fail or be removed is unknown. Fortunately, the SAS procedures that are discussed below are capable of handling right-hand censored data, and in some cases left-hand censored data.

However, in order to develop our empirical distribution, as discussed above, we needed to have an estimate of each failure date. We considered four different approaches to estimating the failure dates:

1. Choose the earliest possible date, which would be the date the retention panel was developed. This was usually one year after the installation.
2. Choose the latest possible date, which would be the date the follow-up survey was completed. This could be anywhere from 2 to 5 years after the installation date.
3. Choose the mid point between the two dates above.
4. Generate a random date between the two dates above, based on the distribution being modeled.

In the end, we selected option four, as it provided us with the smoothest empirical survival function.

Warranties

One other interesting issue is that of warrantied equipment. For this study, failed equipment that is replaced under warranty counted as if it is still operable and in place. It is important to note that equipment is generally replaced under warranty for up to the first five years since installation. Therefore, we should expect the survival function to change significantly after the first five years.

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[3] Please note that a result with a smaller standard error does not indicate that the model is a better fit.

During the first five years the only equipment that would not be in place and operable are those that are removed, or those that fail that are not replaced under warranty.

Confidence Interval Caluculation

The SAS output from the LIFEREG provided the standard errors for the 50th percentile (or median). Because the analysis was conducted on the unit of measure (e.g., a ballast) and not a site, the standard errors from SAS were grossly underestimated. SAS treats each observation in the dataset as independent. However, it is likely that there is significant correlation in the observations that are common to a single site (especially in the event that a removal occurs.) For example, when a removal occurs, it is likely that many measures are removed at once. To a lesser extent, failures are correlated since they may all come from the same manufacturing lot, they are all likely to be installed under the same circumstances, and they are also used in a similar manner.

If we believed that there was 100 percent correlation of failure/removal for all measures with a site, we could simply multiply the standard error calculated from SAS by the square root of the ratio of the number of units to sites. Therefore, if there were an average of 100 units installed per measure, we would multiply by 10.

We felt, however, that there were two components to our error: one caused by variation across sites, and another caused by variation across measures. The errors calculated by SAS correspond only to the error across measures. To address this issue, we adopted the method developed by Skinner[4], who developed an approach to solving the problem of estimating a standard error when the data are not identical and independently distributed (IID). The following is the adjustment that we have made to the standard errors provided by SAS to compute our confidence intervals:

$$StdErr_{Failures,\mathrm{Re}movals} = \sqrt{\frac{0.5*(StdErr_{SAS})^2 * N_{Units}}{N_{Units}} + \frac{0.5*(StdErr_{SAS})^2 * N_{Units}}{N_{Sites}}}$$

$$= StdErr_{SAS} * \sqrt{0.5 + 0.5 * \frac{N_{Units}}{N_{Sites}}}$$

Where,

$StdErr_{Failures,\mathrm{Re}movals}$ is the standard error of the estimated median EUL of failures and removals;

$StdErr_{SAS}$ is the standard error of the median EUL estimated by the SAS procedure;

N_{Units} is the number of units used for the regression models;

N_{Sites} is the total number of sites having those units.

It is interesting to note that if there was only one unit per site, the standard error would equal the standard error calculated in SAS. Our resulting standard error is somewhere between the standard error

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[4] Skinner, C. J., "Analysis of Complex Surveys," John Wiley & Sons, 1989, pp. 23-46.

found in SAS, and the standard error from SAS multiplied by the square root of the ratio of the number of units to sites (the method discussed as the beginning of this section.)

References

Allison, Paul D. 1995 , "Survival Analysis Using the SAS System, A Practical Guide." SAS Institute, NC.

Energy User News, Vol. 23 No. 10, October 1998. Electronics, Energy Products and Life-Cycle Costing: 28.

Skinner, C. J. 1989, "Analysis of Complex Surveys," John Wiley & Sons: 23-46.