

Market Penetration of Competing New Technology: A Maximum Likelihood (MLE) Approach to Modeling the Emergence of the Electronic Ballasts

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ABSTRACT

To study the market penetration of electronic ballasts, the authors started with a conventional Ordinary Least Squares (OLS) approach, tested different functional forms, and corrected for possible failures of the selected model. This study used the Autoregressive Conditional Heteroscedasticity (ARCH) and maximum likelihood estimation (MLE) modeling approach to generate a prediction scheme for electronic ballasts. ARCH has advantages over OLS models for explaining market penetration independent variables over time. Testing for autocorrelation and heteroscedasticity failures is crucial to modeling policy implications and studying impacts of regulatory mandates. The MLE approach has been used to model multiple-product competition for market share between products having extremely long lifetimes and market duration, such as coal or oil. The MLE approach is validated for products such as traditional durable goods of relevance to energy efficiency – in this case of electronic ballasts.

Introduction

Technology is the major driving force of productivity gains and economic growth. Historical studies throughout the last three decades have attributed about half of economic growth to technological change and half to the combined effect of all other driving forces, such as a larger and better-qualified labor force and accumulated stock of capital (e.g. Peterka 1977).

Peterka (1977) extended a variant of the logistic function developed by Fisher and Pry (1971) using maximum likelihood methods. The Peterka model used empirical data on the relative market shares of alternative primary energy sources or fuels (wood, coal, oil, natural gas, and nuclear) over the 1850-1970 period. That model resulted in an algorithm used for modeling the penetration of competing fuels. The authors hope to validate the maximum likelihood technique for application to durable goods of considerably shorter market lifecycles than primary energy sources such as coal or oil.

Background

Prior to the advent of electronic ballasts, fluorescent lamps were powered using magnetic ballasts constructed from inductors, transformers, and capacitors to provide the high voltage needed to start the lamps and control the current at the correct operating level. Electronic ballasts can perform these tasks more efficiently and last longer as well as operate at higher frequencies to further improve the efficiency of the light generation process itself. Through research sponsored by the U.S. Department of Energy (DOE) beginning in 1976, Lawrence Berkeley National Laboratory (LBNL) developed the electronic

ballast technology in use today. Fluorescent light fixtures generally consume 15% to 40% less electricity per unit of light output when an electronic ballast, rather than a magnetic ballast, is used (Geller and McGaragh 1998).

We focused on this relatively narrow component of the lighting fixture market for several reasons:

- In selecting the electronic ballast, the customer must consider the effect the ballast will have on the line power quality. Power quality can be affected by the ballast's power factor (PF), the ratio of (real power/apparent power). Including the power factor variable in the market analysis leads to more practical decision making analysis for the commercial sector customer and the whole sector. Low power factor reduces the electrical system's distribution capacity by increasing current flow and causing voltage drops.
- According to Census data published through the Current Industrial Reports series (Commerce 2001), shipments of corrected power factor magnetic ballasts fluctuated during the 1990s (see Figure 1). The relationship between revenue received by manufacturers and distributors of corrected and uncorrected magnetic ballasts, as an explanatory variable, and the market share of electronic ballasts, as a dependent variable, has not been estimated previously. Furthermore, electronic ballasts had captured ~50% of the \$1 billion ballast market by 2001 (Commerce 2001).
- Electronic ballast market penetration throughout the last three decades provides a case study that could be applied to other lighting technologies which are already commercially available in some form, or which are nearing commercial availability. Examples of new technologies include solid-state lighting (LED traffic signals), High Intensity Discharge (HID) lamps etc.
- DOE (1995) estimated that the use of electronic ballasts saved consumers ~\$750 million in energy bills for the period from 1987-1995. The electronic ballast market share in 1995 was ~30% of the ballast market. Using the DOE (1995) estimate, on average every 1% increase in market share of electronic ballast generates ~ \$25 million in consumer surplus.

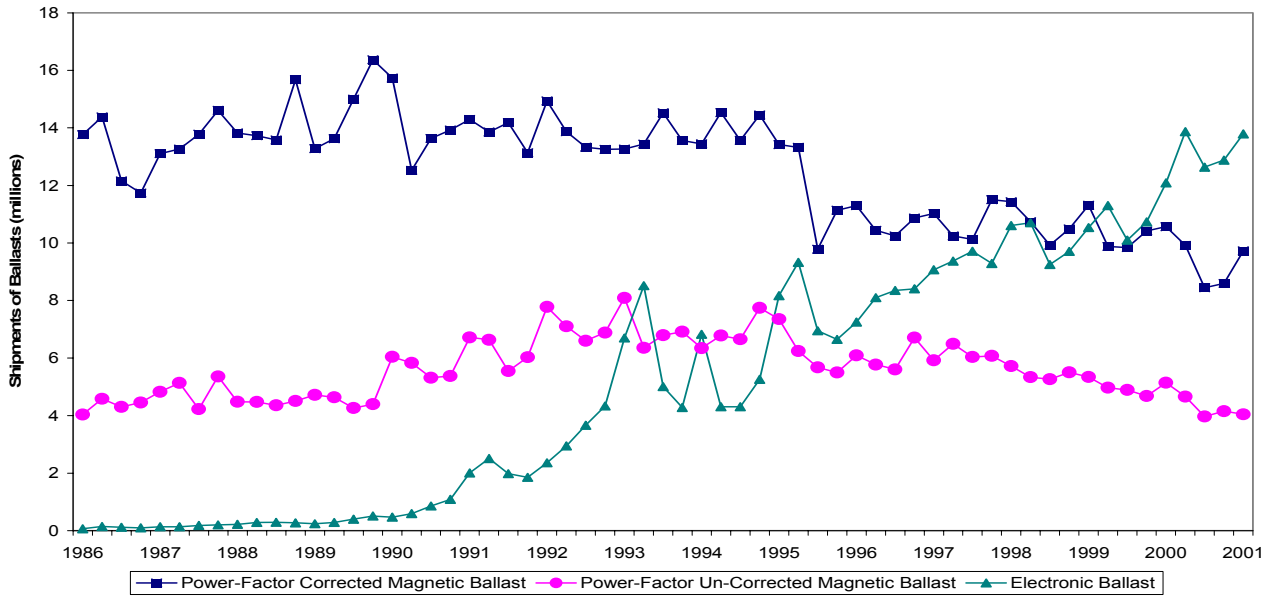


Figure 1. Quarterly Shipments of Domestically-Produced Ballasts to the U.S. Commercial Sector

Method

To study the market penetration of competing technologies econometrically, the authors started with conventional Ordinary Least Squares (OLS); tested the validity of the linear, semi-log, and log-log functional forms; and corrected for possible failures of the selected model. Possible causes of model failure include the violation of the basic assumption of no autocorrelation between the disturbances, violation of the equal variance of these disturbances, and specification errors. Correction for specification error is considered by detecting the presence of unnecessary variables or incorrect function form (see the Appendix).

ARCH models were introduced by Engle (1982) and were generalized as Generalized ARCH (GARCH) by Bollerslev (1986). These models are widely used in various branches of econometrics, especially in financial time series analysis. See Bollerslev, Chou, and Kroner (1992); Bollerslev, Engle, and Nelson (1994); Hamilton (1994); Gujarati (1995); and Greene (1997) for relevant surveys. The ARCH model offers a solution for both autocorrelation and heteroscedasticity problems. Gujarati (1995) summarized the ARCH as follows:

The key idea of the ARCH model is that the variance of ε at time t ($=\sigma_t^2$) depends on the size of the square error term at time $(t-1)$, that is, on ε_{t-1}^2 ,

$$Y_t = \beta_1 + \beta_2 X_{2t} + \dots + \beta_k X_{kt} + \varepsilon_t \dots \dots \dots (1)$$

and assumes the conditional on the information variable at time $(t-1)$, the disturbance term is distributed as $\varepsilon_t \sim N[0, (\alpha_0 + \alpha_1 \varepsilon_{t-1}^2)$

Thus the ARCH(ρ) process can be written as:

$$\text{Var}(\varepsilon_t) = \delta_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 \dots\dots\dots(2)$$

ARCH models are specifically designed to model and forecast conditional variances. The variance of the dependent variable is modeled as a function of past values of the dependent variable and independent or explanatory variables.

A concept frequently used in econometrics is that of Maximum Likelihood Estimation (MLE). MLE stemmed from the idea that different statistical populations generate different samples; β^{MLE} is simply the value of β that maximizes the probability of drawing the sample actually obtained. Goldfield and Quandt (1972) concluded that maximum likelihood techniques perform well in a wide variety of applications and for relatively small sample sizes. It is particularly evident from their book that the MLE is well suited to estimation involving nonlinearities and unusual estimation problems such as the expected “S” shape of technology penetration. MLE estimates the likelihood function under the assumption that the contemporaneous errors have a joint normal distribution.

The maximum likelihood function used can be represented as follow:

$$L(\beta) = \prod_i \{1 - V_i[-\beta' X(t)]\} \prod_i V_i[-\beta' X(t)] \quad t = \text{from } 1986-01_to_2001-01 \dots\dots\dots(3)$$

where V_i is the distribution of ε_i and X is a vector of explanatory variables.

The ARCH formulation is a function of the lagged values of the model explanatory variables as well as the squares and cross products of lagged forecast errors. Although the analytic derivations of (3) can be computed (See Engle et al. 1987), variable-metric algorithms which employ numerical derivatives are simpler to use and easily allow changes in specification. Under suitable regularity conditions, maximization of (3) will yield maximum likelihood estimates with the Best Linear Unbiased Estimates (BLUE) properties. Pindyck and Rubinfeld (1981) have shown that “the maximum-likelihood estimators of α and β are identically equal to the least-square estimators.” It follows therefore that α' and β' are BLUE, but $\delta^{2'}$ however, is a biased (although consistent) estimator of δ^2 .

This econometric property of the MLE makes it a viable tool for experiments and testing against the OLS in practical applications.

Results

In the following section, we discuss the results of both the OLS and ARCH model based on MLE models and compare the results obtained.

A: The Ordinary Least Square Model

The designed model assumes that market share of the electronic ballast is a dependent variable of a set of economic and policy explanatory variables. The explanatory variables include some of both economic and policy market factors. The economic variables include the electricity price at the commercial sector level and the revenue generated by each of the competing technologies (power-

factor-corrected magnetic ballast), and the commercial sector new construction buildings permits. A dummy for the existence of fluorescent ballast standards provides a policy variable. Data on quarterly electronic and magnetic ballast quantity of shipments, revenues to manufacturers and distributors, and commercial-sector new construction permits data were collected from the Census Bureau's current industrial reports for the period from the first quarter of 1986 to the first quarter of 2001. Electricity prices were obtained from the DOE's Energy Information Administration (EIA, 2002) for the same time period. The dummy variable for fluorescent lamp ballast standards is introduced to the model beginning in 1990, the year the efficiency standards initially became effective.

Figure (2) shows actual, fitted, and residual values from the OLS model. Results of the untransformed OLS linear model of electronic ballast market penetration showed that all specified explanatory variables are significant at 0.05 levels except for the commercial sector electricity prices, and the power-factor-uncorrected magnetic ballast (t-statistics in Table 1). The signs of the significant explanatory variables seem to correspond to the expected economic logic of each of the variables except for the power-factor-uncorrected magnetic ballast. The negative sign of the power-factor-corrected magnetic ballast indicates the negative relationship between this variable and the electronic ballast market share. The fluorescent-ballast-standard dummy variable, which takes 0 for all periods prior to the standard being enacted (1990), and 1 afterward is significant and positively impacts the electronic ballast market share. Both the new commercial construction permits and the electricity price at the commercial level indicated a positive relationship between these explanatory variables and the dependent variable. However, the electricity price is not significant, which confirms the fact that the model is facing a misspecification problem. The power-factor-uncorrected variable sign and insignificance results suggest model misspecification and the possibility of one of the other three potential failures of such a model: autocorrelation, heteroscedasticity and multicollinearity.

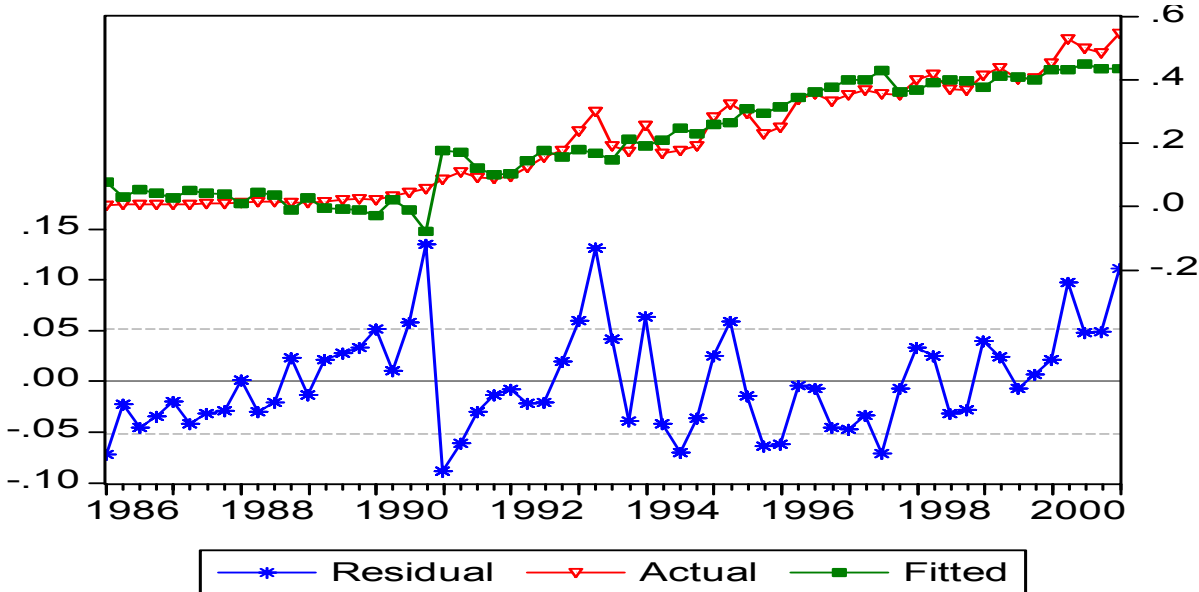


Figure 2. Electronic Ballast Market Share Ordinary Least Square Base Model

Table 1. Linear Ordinary Least Square Regression of Electronic Ballast Market Share

| Variable | Coefficient | Std. Error | t-Statistic | P-Value |
|--|-------------|-----------------------|-------------|----------|
| Constant | -0.4013 | 0.258023 | -1.555277 | 0.125600 |
| Power-Factor-Corrected Magnetic Ballast Revenue | -0.000002 | 0.000001 | -3.336945 | 0.001500 |
| Power-Factor-Un-Corrected Magnetic Ballast Revenue | 0.000005 | 0.000005 | 1.006103 | 0.318800 |
| Fluorescent Ballast Standards New Commercial Constructions Permits | 0.278597 | 0.037488 | 7.431705 | 0.000000 |
| Commercial Sector Electricity Price | 0.000004 | 0.000001 | 6.149038 | 0.000000 |
| | 0.014229 | 0.029952 | 0.475059 | 0.636600 |
| R-squared | 0.9149 | Mean dependent var | 0.205123 | |
| Adjusted R-squared | 0.9071 | S.D. dependent var | 0.16996 | |
| S.E. of regression | 0.0518 | Akaike info criterion | -2.990037 | |
| Sum squared resid | 0.1475 | Schwarz criterion | -2.78241 | |
| Log likelihood | 97.1961 | F-statistic | 118.2332 | |
| Durbin-Watson stat | 1.1245 | Prob(F-statistic) | 0 | |

Furthermore the OLS showed a high R^2 (0.91) and a low Durbin-Watson (1.12) which suggest the possibility of a model encountering an autocorrelation failure. The model residual line in Figure (2) and the correlogram plot in Figure (3) detected the possibility of a model failure caused by an autocorrelation between the disturbance terms. Except for the shock introduced to the model in 1990 to represent the fluorescent ballast standards, the model's disturbance had an upward linear trend prior to 1992 and a general downward trend for the 1992-1998 period. Figure (1) showed an upward trend in the residuals from 1986 to roughly 1990; then a sharp discontinuity; followed by another upward trend to roughly 1993; followed by a downward trend from 1993 to 1997; followed by a clear upward trend from 1997 onward. This pattern of upward and downward changes in the plotted residuals indicated a possibility of a positive autocorrelation between the current and lagged error terms.

To detect and statistically confirm the presence autocorrelation, the Breusch-Godfrey (BG) test of higher-order autocorrelation was used. The BG test assumes that the disturbance term μ_t is generated by the following ρ th-order autoregressive scheme:

$$\mu_t = \rho_1 \mu_{t-1} + \rho_2 \mu_{t-2} + \dots + \rho_p \mu_{t-p} + \varepsilon \quad \dots \dots \dots (4)$$

The other notation is described in equations (1) and (2).

The BG null hypotheses H_0 is: $\rho_1 = \rho_2 = \dots = \rho_p = 0$. Breusch and Godfrey test has shown that $(n-p)R^2 \sim \chi_p^2 = 10.42991$, with a corresponding P-value of 0.005435. The small P-value of the BG test is another evidence of an autocorrelation between the disturbance terms of the OLS model.

To detect the presence heteroscedasticity, the Breusch-Pagan (BP) test of homoscedasticity was used. A Wald test on the (BP) was implemented to test the null hypothesis that; $H_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5$. The P-Value correspondent to the Chi square of the test found to be 0.1989

The Wald test result confirmed the presence of heteroscedasticity and the null hypothesis was rejected and the presence of heteroscedasticity was confirmed.

B: Autoregressive Conditional Heteroscedasticity (ARCH) Maximum Likelihood Model

Engle (1982) developed the ARCH model to correct for an autocorrelation because the residual variance $\hat{\sigma}^2$ in an OLS case may underestimate the true variance. Disregarding autocorrelation results in overestimation of R^2 , the usual t and F tests of significance are no longer valid. Figure (3) below shows the actual fitted and residual plot of the ARCH model of the electronic ballast market share. The downward or upwards patterns in the disturbance terms disappeared as compared to the OLS in Figure (2) and stayed primarily within the region of -0.1 to 0.1 market shares.

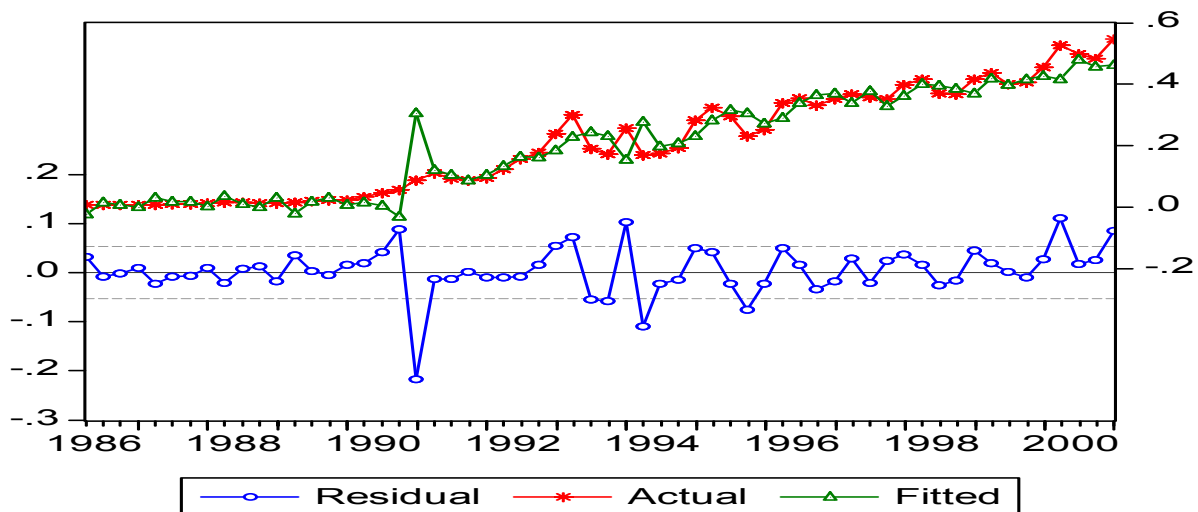


Figure 3. Electronic Ballast Market Share ARCH Model

Results of the base ARCH model of electronic ballast market penetration are presented in Table (2). Introducing the ARCH model kept the R^2 at 0.92 but enhanced the Durbin Watson stat to ~ 2 , which means the ARCH model helped solve the problem of autocorrelation between the disturbance terms. Other results showed that all specified explanatory variables are significant at 0.05 levels except for power-factor-un-corrected magnetic ballast (t-statistics in Table 2), which still showed insignificant t-statistics. The signs of all explanatory variables now seem to correspond more closely to the expected economic logic of each of the variables, including the power-factor-uncorrected magnetic ballast.

The negative signs of the power-factor-corrected and power-factor-uncorrected magnetic ballast indicate the negative relationship between these two variables and the electronic ballast market share. This result confirmed the insignificant effect of the power-factor-uncorrected magnetic ballast. The power-factor-corrected magnetic ballast revenue increases by $\sim \$1$ million, the market share of electronic ballast increases by $\sim 1\%$. The t statistic of the corrected magnetic ballast is significant at the 0.05 level of significance, which suggests that the corrected magnetic ballast and electronic ballast vie for the same segment of the ballast market. The power-factor-uncorrected magnetic ballast coefficient is not significant because the electronic ballast faces larger competition from the corrected one. The fluorescent-ballast-standard dummy variable is highly significant and positively impacts the electronic

ballast market share. The new-commercial-construction-permits variable indicated a positive relationship between the explanatory variable and the dependent variable, however it has a smaller influence on the dependent variable relative to that of fluorescent ballast standards. The electricity price is significant, which confirms that the commercial sector is responsive to the electricity prices

Performing the ARCH Q-statistic correlogram test showed that introducing the ARCH scheme produces a random disturbance of the disturbance terms (see also Figure 3 above).

Table 2. ARCH Regression of Electronic Ballast Market Share

| Variable | Coefficient | Std. Error | t-Statistic | P -Value |
|--|--------------------|-----------------------|--------------------|-----------------|
| Constant | -0.265147 | 0.067783 | -3.911683 | 0.0001 |
| Power-Factor-Corrected Magnetic Ballast Revenue | -0.000001 | 0.000000 | -3.701464 | 0.0002 |
| Power-Factor-Un-Corrected Magnetic Ballast Revenue | -0.000002 | 0.000001 | -1.600157 | 0.1096 |
| Fluorescent Ballast Standards | 0.338484 | 0.013457 | 25.153290 | 0 |
| New Commercial Constructions Permits | 0.000003 | 0.000000 | 8.769769 | 0 |
| Commercial Sector Electricity Price | -0.017022 | 0.007477 | -2.276670 | 0.0228 |
| AR(1) | 0.647036 | 0.067324 | 9.610752 | 0 |
| R-squared | 0.915291 | Mean dependent var | | 0.208484 |
| Adjusted R-squared | 0.902004 | S.D. dependent var | | 0.169338 |
| S.E. of regression | 0.05301 | Akaike info criterion | | -3.4626 |
| Sum squared resid | 0.143314 | Schwarz criterion | | -3.148448 |
| Log likelihood | 112.878 | F-statistic | | 68.88289 |
| Durbin-Watson stat | 2.119764 | Prob(F-statistic) | | 0 |
| Inverted AR Roots | 0.65 | | | |

Conclusion

This study demonstrated that using the ARCH model and MLE approach has advantages over standard OLS models for explaining electronic ballast market penetration over time. Testing for autocorrelation and heteroscedasticity failures proved crucial to modeling policy implications and studying impacts of regulatory mandates.

The success of these techniques suggests that algorithms such as those developed by Peterka (1977) can be applied to products such as durable goods having considerably shorter market life-cycles than, say, the evolution of fuels for primary energy. These results apply to energy-efficient products such as electronic ballasts and other measures used to reduce energy consumption. This means that algorithms developed using MLE techniques for modeling product market penetration under competition should be considered a valid approach.

References

- Bass, F. (1969), "A new product growth model for consumer durables." *Management Science*, 15 (5), 215-227, Institute for Operations Research and the Management Sciences, Linthicum, MD.
- Bertrand, M. and S. Mullainathan (2001). Do People Mean What they Say? Implications for Subjective Survey Data. *American Economic Review Papers and Proceedings* Volume 91. Number 2 PP.67-72.
- Bollerslev, Tim (1986). Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31, 307-327.
- Bollerslev, Tim, Ray Y. Chou, and Kenneth F. Kroner (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence, *Journal of Econometrics*, 52, 5-59.
- Bollerslev, Tim, Robert F. Engle and Daniel B. Nelson (1994). ARCH Models, Chapter 49 in Robert F. Engle and Daniel L. McFadden (eds.), *Handbook of Econometrics*, Volume 4, North-Holland.
- DOE, Office of Science Policy. (1995) DOE Successful Stories: The Energy Mission in the Marketplace.
- DOE, Office of Building Technologies (OBT) (October, 1999). Energy Efficiency and Renewable Energy (EERE). Energy Conservation Standards for Consumer Products: Fluorescent Lamp Ballasts.
- Duke, Richard and Daniel M. Kammen (1999). The Economics of Energy Market Transformation Programs. *The Energy Journal*, 20(4): 15-64.
- Engle, Robert F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation, *Econometrica*, 50, 987-1008.
- Fisher, J.C., and R.H. Pry, (1971) "A Simple Substitution Model of Technological Change." *Technological Forecasting and Social Change*, 3, 75-88.
- Geller, H. and S. McGaragh. (1998). Successful Government – Industry Partnership: the U.S. Department of Energy's Role in Advancing Energy-Efficient Technologies. *Energy Policy* Vol. 26 No. 3 PP 167-177.
- Greene, William H. 1997. *Econometric Analysis*. Prentice Hall. New Jersey.
- Goldfield, S. and R. Quandt (1972). *Nonlinear Methods in Econometrics*. Amsterdam, North Holland.
- Gujarati, Damodar N. 1995. *Basic Econometrics*. McGraw-Hill Inc. 3rd ed.
- Hamilton, James D. (1994). *Time Series Analysis*. Princeton University Press. Princeton, New Jersey.
- Peterka V. and F. Fleck 1977. *The Dynamic of Energy Systems and Logistic Substitution Model. Vol. 2: Theoretical Part*. International Institute for Applied Systems analysis.

Pindyck, Robert S. and Daniel L. Rubinfeld. 1981. *Econometric Models and Economic Forecast*. McGraw-Hill.

Commerce, 2001, Current Industrial Reports. Fluorescent Lamp Ballasts. 1986-2000, Census Bureau, Economic and Statistics Administration. U.S. Department of Commerce.

Vorsatz, Diana, Leslie Shown, Jonathan G. Koomey, Mithra Moezzi, Andrea Denver, and Barbara Atkinson. 1997. Lighting Market Sourcebook. Ernest Orlando Lawrence Berkeley National Laboratory. LBNL-39102. December. <http://enduse.lbl.gov/Projects/LMS.html>

Appendix: Preliminary Models
Log-log Regression of Electronic Ballast Market Share

| Variable | Coefficient | Std. Error | t-Statistic | P-Value |
|--|--------------------|-----------------------|--------------------|----------------|
| Constant | -56.320860 | 12.027460 | -4.682691 | 0.000000 |
| Power-Factor-Corrected Magnetic Ballast Revenue | -0.285756 | 0.584576 | -0.488826 | 0.626900 |
| Power-Factor-Un-Corrected Magnetic Ballast Revenue | 1.815095 | 0.650323 | 2.791068 | 0.007200 |
| Fluorescent Ballast Standards | 2.301774 | 0.345194 | 6.668057 | 0.000000 |
| New Commercial Constructions Permits | 2.640530 | 0.596866 | 4.423994 | 0.000000 |
| Commercial Sector Electricity Price | 3.601355 | 1.892912 | 1.902548 | 0.062300 |
| R-squared | 0.923488 | Mean dependent var | -2.307768 | |
| Adjusted R-squared | 0.916532 | S.D. dependent var | 1.522502 | |
| S.E. of regression | 0.439863 | Akaike info criterion | 1.288476 | |
| Sum squared resid | 10.641390 | Schwarz criterion | 1.496102 | |
| Log likelihood | -33.298500 | F-statistic | 132.767700 | |
| Durbin-Watson stat | 0.887219 | Prob(F-statistic) | 0.000000 | |

Semi-log Regression of Electronic Ballast Market Share (Dependent Variable Only)

| Variable | Coefficient | Std. Error | t-Statistic | P-Value |
|--|--------------------|-----------------------|--------------------|----------------|
| Constant | -13.208530 | 2.216413 | -5.959414 | 0.000000 |
| Power-Factor-Corrected Magnetic Ballast Revenue | 0.000008 | 0.000007 | 1.087979 | 0.281300 |
| Power-Factor-Un-Corrected Magnetic Ballast Revenue | 0.000074 | 0.000041 | 1.811107 | 0.075600 |
| Fluorescent Ballast Standards | 2.764661 | 0.314608 | 8.787636 | 0.000000 |
| New Commercial Constructions Permits | 0.000026 | 0.000005 | 5.775647 | 0.000000 |
| Commercial Sector Electricity Price | 0.511494 | 0.260362 | 1.964545 | 0.054500 |
| R-squared | 0.92001 | Mean dependent var | -2.307768 | |
| Adjusted R-squared | 0.912738 | S.D. dependent var | 1.522502 | |
| S.E. of regression | 0.449749 | Akaike info criterion | 1.332925 | |
| Sum squared resid | 11.12506 | Schwarz criterion | 1.540552 | |
| Log likelihood | -34.65421 | F-statistic | 126.5173 | |
| Durbin-Watson stat | 0.944855 | Prob(F-statistic) | 0 | |

Semi-log Regression of Electronic Ballast Market Share (Explanatory Variables Only)

| Variable | Coefficient | Std. Error | t-Statistic | P-Value |
|--|--------------------|-----------------------|--------------------|----------------|
| Constant | -3.160648 | 1.428818 | -2.212072 | 0.031100 |
| Power-Factor-Corrected Magnetic Ballast Revenue | -0.231166 | 0.065683 | -3.519409 | 0.000900 |
| Power-Factor-Un-Corrected Magnetic Ballast Revenue | 0.049420 | 0.071046 | 0.695597 | 0.489600 |
| Fluorescent Ballast Standards | 0.276204 | 0.039992 | 6.906420 | 0.000000 |
| New Commercial Constructions Permits | 0.441868 | 0.067206 | 6.574827 | 0.000000 |
| Commercial Sector Electricity Price | 0.047850 | 0.227361 | 0.210460 | 0.834100 |
| R-squared | 0.911599 | Mean dependent var | | 0.20512 |
| Adjusted R-squared | 0.903562 | S.D. dependent var | | 0.16996 |
| S.E. of regression | 0.05278 | Akaike info criterion | | -2.95218 |
| Sum squared resid | 0.153216 | Schwarz criterion | | -2.74456 |
| Log likelihood | 96.04161 | F-statistic | | 113.433 |
| Durbin-Watson stat | 1.165632 | Prob(F-statistic) | | 0 |