Measurement Uncertainty and Risk in Measurement and Verification Projects

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Abstract

The relative contribution of measurement uncertainty to combined measurement and sampling uncertainty is investigated in the context of Measurement and Verification (M&V) projects where the whole population is not metered. An example of an M&V energy meter conforming to the 0.5S accuracy class is considered. Using normal distribution statistics and realistic ranges for the relevant parameters, it is found that measurement uncertainty makes a negligible contribution to the overall uncertainty for electricity metering cases where population variance is not unusually low. The case of instruments other than electricity meters, such as thermocouples, is considered, and it is found that in such cases measurement uncertainty may make a material contribution to overall uncertainty. The relationship between energy project risk and M&V meter accuracy is defined and explored by using the example of a single metered facility. It is found that installing higher accuracy meters does not guarantee a decrease in project risk in performance contracting situations where guaranteed or shared savings models are adopted. These methods and results are useful for energy professionals when making metering decisions in the context of project cost and risk.

Energy Use Measurement Uncertainty in M&V

Three kinds of uncertainty have been identified in the M&V context: measurement uncertainty, sampling uncertainty, and modelling uncertainty. To date, most work has focussed on sampling uncertainty and modelling uncertainty. Measurement uncertainty has been assumed to be negligible. In this paper, we will investigate the contribution of measurement uncertainty relative to sampling uncertainty in the M&V context, establishing limits to when this assumption holds, and how these numbers influence project decisions.

From the outset it should be noted that the relative contribution of measurement uncertainty can only be considered when there are other sources of uncertainty, such as sampling. In cases where all facilities are metered or revenue meter data are used, measurement uncertainty is the only source of uncertainty, other than that arising from the mathematical model. The trade-off between installing higher or lower accuracy meters in such a case is explored in the section 'Meter Selection and Risk Mitigation'.

Although the project risk associated with measurement uncertainty has been identified by both researchers and practitioners (Lee, Lam & Lee 2015), it has not been addressed in literature, to our knowledge. Related investigations have been performed, for example in the American Society for Heating, Refrigeration and Air Conditioning Engineers' (ASHRAE) Guideline RA96:

Engineering Analysis of Experimental Data (ASHRAE 1986), a comprehensive introduction to handling uncertainty in engineering measurements is provided. ASHRAE Guideline 14-2002: Measurement of Energy and Demand Savings (ASHRAE 2002) catalogues the accuracies for different measurement equipment used in this study, and also contains further methods for calculation. Regarding financial decision support for Energy Performance Contracting (EPC), risk has been analysed from an economic perspective using Monte Carlo analysis (Jackson 2010) and the US Department of Energy's *EnergyPlus* software (Lee et al. 2013). Deng et al. (2015) provide a useful summary of the design of energy performance contracts under uncertainty. Bayesian methods have also been implemented for building simulation covariate calibration and uncertainty analysis (Heo, Choudhary & Augenbroe 2012; Heo & Zavala, 2012), and following from that, quantitative risk analysis for decision support in retrofit project planning was explored (Heo, Augenbroe & Choudhary 2013). These studies focus on simulation accuracy rather than metering decision making. Finally, some of the most relevant research on this topic is in the area of legal metrology, where measurement uncertainty and cost are traded off in a decision support framework (Pendrill & Källgren 2006). The focus of their study is conformance to a given standard rather than the verification of individual measurements, and risk was viewed from a government perspective as a function of the cost to society.

Governance Structure

In the South African context, an M&V Team is an independent third party verifying the savings achieved by the Energy Services Company (Esco) also called the Project Developer (PD), on behalf of a client or facility owner. These energy efficiency or demand side management projects are usually incentivized by the national electricity utility, Eskom, or initiatives such as the United Nations Clean Development Mechanism. Measurement and Verification with reporting inside given uncertainty bounds is usually stipulated in the contract.

Meters can be purchased by the sponsor, Eskom, although for private projects or to expedite utility projects, the client often purchases meters. Examples in this paper will assume such governance structures.

Paper Outline

In this paper, we will first devise a method for incorporating measurement uncertainty into the standard M&V measures of reporting uncertainty and sample size calculations. These relations are especially useful to M&V professionals, but can be used by project developers as well. They are implemented in a number of examples for the rest of the paper. The first is the question of choosing between two common accuracy classes of electricity meter: 0.2S and 0.5S. A next case considers cheap energy meters using a clip-on current transformers are installed. What accuracy can we expect, given unmeasured voltage fluctuations? This is extended even further to other measurement instruments, and complexities are discussed. Last, a case study is presented related to whether a project developer in a guaranteed savings energy performance contract can derive any monetary advantage from installing a more accurate (and more expensive) meter.

Calculation Method

Before a detailed investigation of measurement uncertainty can be made, the sampling distribution should be carefully defined. There are three distributions relevant to sampling: The *population distribution* is the true distribution of the population, and is unavailable to the engineer unless he samples the total population with perfect measurement equipment. The *sampling distribution* is the theoretical distribution for samples of a given size. With perfect measurement equipment, the sample distribution will be equal to the sampling distribution. The *sample distribution* is the observed distribution on the sample that was actually taken, with the measurement equipment actually used. This is the only distribution accessible to the engineer.

The calculations below only are valid under the standard statistical assumptions of independent, normally distributed data. We also assume that although the measurement instrument may be inaccurate, a large population of such instruments will be unbiased. This implies that the measured sample mean will tend to the true mean as the sample size tends to infinity. We also assume that measurement errors are normally distributed around the mean.

Let the subscript $_s$ denote the (theoretical) sampling distribution, and the subscript $_m$ denote measurement parameters. Furthermore, let σ_m be the measured standard deviation of the sample and z_m be the standard score of the known confidence level α on the measured data. Since only measured data is available, consider s_m as the sample standard deviation and $\bar{x}$ as the sample mean, and p_m as the precision or error bound. The upper limit of this error bound should be equal to the upper confidence limit:

$$\bar{x} + p_m\bar{x} = \bar{x} + s_m z_m, \tag{1}$$

$$\therefore\ p_m\bar{x} = s_m z_m, \tag{2}$$

$$\therefore\ s_m = \frac{p_m}{z_m}\bar{x}. \tag{3}$$

The standard deviation, and therefore the variance and distribution on the measurement data has now been characterised by writing the standard deviation in terms of the known precision level, desired confidence level, and the sample mean.

Although this is common knowledge to most practitioners, it is necessary to clarify the different ways in which errors may be expressed in Measurement and Verification (M&V). First, the error of some measurement system may be expressed statistically as a standard deviation from the mean, or it may be expressed as a maximum error. The maximum error approach is popular and conservative. However, it represents a highly unlikely and unnecessarily strict case where all the individual errors are assumed to be at their maxima simultaneously. We will consider the statistical approach. The total error is calculated as a root mean square, which the way in which standard deviations are added. It should also be stated with a certain confidence level.

Errors can also be expressed in absolute or relative terms. 200 kWh±10 kWh has an absolute error of 10 kWh, but a relative error of 5%. The expressions for adding and multiplying uncertain values differ according to which expression is used. We will be expressing errors in relative (percentage) terms, except when stated otherwise.

When combining two independent normal distributions, the means are added arithmetically. However, the total variance of the combined distribution should be

$$s^2_{combined} = s^2_s + s^2_m, \tag{4}$$

where s^2_s is the sampling variance. But from (3),

$$s^2_m = \frac{p^2_m}{z^2_m}\bar{x}^2. \tag{5}$$

Therefore,

$$s^2_{combined} = s^2_s + \frac{p^2_m}{z^2_m}\bar{x}^2. \tag{6}$$

It is useful to define these relations in terms of the coefficient of variance(CV), since this makes the calculation independent of the size of the mean and variance:

$$\mathrm{CV} = \frac{s}{\bar{x}}. \tag{7}$$

Also, since sample size required for M&V reporting is proportional to the CV value, the relative contribution of measurement uncertainty to $\mathrm{CV}_{combined}$ is an indication of size of the effect of measurement uncertainty on overall project cost. Substituting (6) we can now define the combined CV as

$$\mathrm{CV}_{combined} = \frac{s_{combined}}{\bar{x}} = \frac{\sqrt{s^2_s + \frac{p^2_m}{z^2_m}\bar{x}^2}}{\bar{x}}. \tag{8}$$

This may be further simplified to

$$\mathrm{CV}_{combined} = \sqrt{\mathrm{CV}^2_s + \frac{p^2_m}{z^2_m}}. \tag{9}$$

We have now reduced the combined CV to a formula needing only values that are readily available (meter accuracy), and widely estimated (CV_s). An example of (9) is plotted in Figure 1 at the 95% confidence level, which is the most common one used in metrology (ASHRAE 1986). This corresponds to a "coverage factor" of $k = 2$, or 2σ. The coverage factor refers to the area under the normal distribution curve, similar to a two-sided confidence interval, expressed in terms of standard deviations, or "sigmas". So a coverage factor of one corresponds to 1σ, or 68% confidence. A coverage factor of two would be 2σ or 95.44%, a coverage factor of 3 to 99.74%, and so forth. We can see that for $p_m \leq 0.1$ and $\mathrm{CV}_s \geq 0.2$, the overall uncertainty is dominated by sampling uncertainty, and measurement uncertainty can be safely neglected.

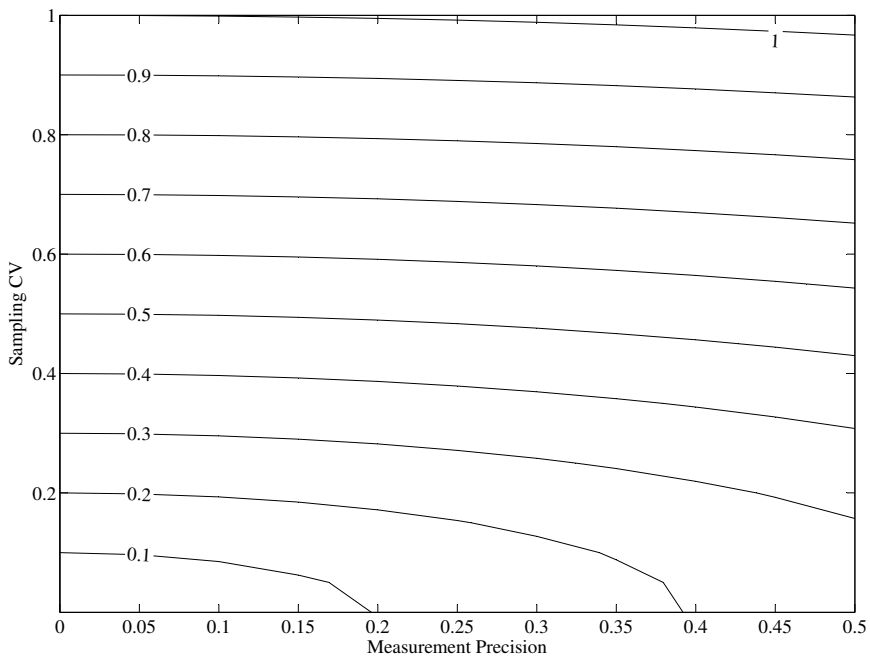

Figure 1: Contour plot of combined CV as a function of sample CV and measurement precision in equation (9), for a measurement confidence of 95%.

The most common reporting level is "90/10", that is, the reporting confidence is 90%, and the reporting precision 10%. The formula for the sample size n required to report with a given confidence α_r at z_r, and a precision p_r, is:

$$n = \frac{z_r^2 \mathrm{CV}^2}{p_r^2}. \tag{10}$$

By substituting (9), we can write the required sample size as a function of sampling CV, and measurement accuracy, and required reporting precision:

$$n = \left(\mathrm{CV}_s^2 + \frac{p_m^2}{z_m^2}\right)\frac{z_r^2}{p_r^2}. \tag{11}$$

An example of (11) is plotted in Figure 2.

Practical Implementation

Realistic case. Consider the case of M&V (not revenue) energy meters that conform to the ANSI C12.20 standard (ANSI 2002), and its international counterpart, the IEC 62053-22 (IEC 2003). These standards specify that electricity meters should have an accuracy of 0.5% for class 0.5S and 0.2% for class 0.2S during normal operation. However, for the 0.5S class, precision may be up to 1% for low power factors. ASHRAE 14-2002 Technical note #7 of A5.6.2.1 (ASHRAE 2002) gives the instrument system error as 2%, which includes the CT accuracy. The standards do not specify a confidence interval on these values. We may therefore select the 2% value as a realistically low precision, and $z_m = 1.96$, which corresponds to an 95% confidence

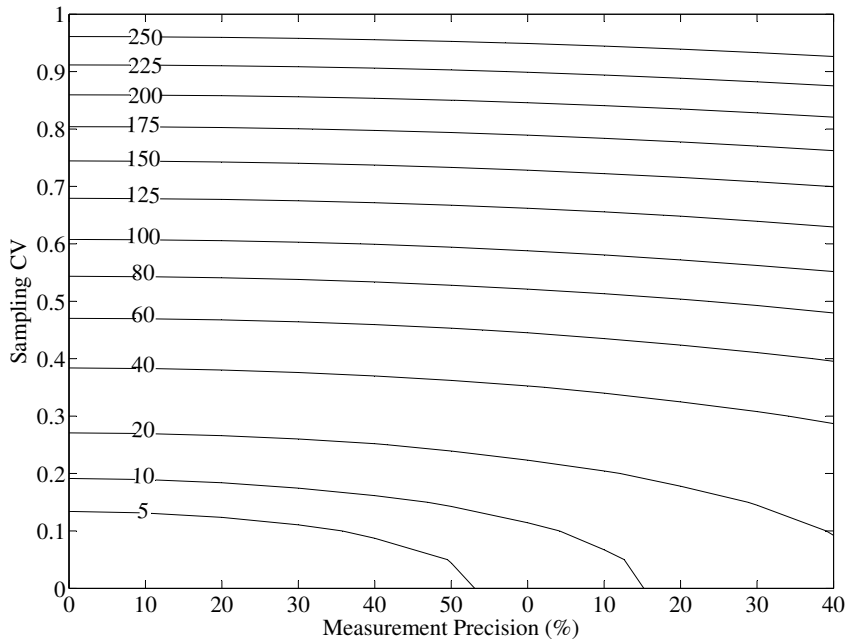

Figure 2: Contour plot of (11): The required sample size as a function of sampling CV and measurement precision for reporting at the 90/10 level.

level (ASHRAE 1986). We may also assume CV= 0.05: that is, a stable process with a coefficient of variance of 5%. This reduces the contribution of sampling uncertainty to overall uncertainty.

Using the above-mentioned figures, by (9), the ratio between $\mathrm{CV}_{combined}$ and CV_s is:

$$\frac{\mathrm{CV}_{combined}}{\mathrm{CV}_s} = \frac{\sqrt{\mathrm{CV}_s^2 + \frac{p_m^2}{z_m^2}}}{\mathrm{CV}_s}. \tag{12}$$

Therefore, by substituting our assumptions above,

$$\frac{\mathrm{CV}_{combined}}{\mathrm{CV}_s} = \frac{\sqrt{0.05^2 + \frac{0.02^2}{1.96^2}}}{0.05} = 1.021. \tag{13}$$

Thus in a worst-case scenario, measurement uncertainty would add 2.1% relative to the sampling uncertainty. In other words, if the uncertainty on the savings is 10%, sampling uncertainty comprises roughly 9.8% of this figure, and measurement uncertainty comprises the other 0.2%. It can be seen that in such cases, measurement uncertainty may be neglected in most practical meter sampling applications such as Residential Mass Rollout (RMR) projects.

Case of Supply Voltage Unknown. It should be noted that certain cheaper meters use split-core current transformers (CT's) clipped around the line, and do not measure voltage. These meters are a considerably more cost-effective alternative in terms of meter cost and installation complexity. However, for loads where voltage isn't regulated by appliance circuitry, such as resistance heating equipment, energy use would be directly proportional to the unmeasured voltage.

The true measurement uncertainty is then much higher than that reported above. In Europe, utility supply voltage is determined to be 230V±10% (CENELEC 1988), and in the United States, 120V±5% (ANSI 2006). However, certain asymmetrical tolerances may also hold. For ANSI C84.1 Range B, these tolerances are −13% and +6%. These asymmetrical tolerances may skew the calculation, since under-voltages are twice as likely as over-voltages, and equipment may therefore consume less energy than reported by the meter.

For the symmetrical tolerance case of ±10%, if voltage variation were Gaussian with a mean at the nominal voltage, these variations would cancel out over time. However, the supply voltage at a facility such as a house varies according to a number of factors. For example, it varies with the distance of its distribution transformer from the substation on the primary feeder, the distance between this house and the transformer on the secondary feeder, the number of facilities on the secondary feeder, etc. Therefore the distribution of voltage is not Gaussian and will not cancel out over time. In order to simplify most large-scale metering projects, however, it may still be assumed that the measurement error is Gaussian, since if you sample enough facilities over a large area, some will be close to the transformers and some will be further away. For a small sample over a small area of the grid, or where facilities fall along a limited number of branches of a distribution network, this simplifying assumption will not hold, and supply voltage should be measured.

If we assume that the service level of the utility with respect to the voltage tolerance is 99%, we can calculate the total measurement error. However, we first have to convert the 99/10 accuracy specification to the $95/p_2$ level in order to do the calculation.

From the standard normal formula, for x as the unknown point voltage, μ as the mean supply voltage, and σ as the standard deviation on the supply voltage,

$$z = \frac{x - \mu}{\sigma}. \tag{14}$$

Now, for $\alpha_1 = 99\%, z_1 = 2.576$ and $\alpha_2 = 95\%, z_2 = 1.96$. Let x_1 be the upper tolerance limit of p_1 =10% on 230V, therefore $x_1 = 253$V. Let x_2 be the unknown voltage limit at $\alpha = 95\%$ (the instrument confidence level). We need to convert the supply voltage error specification to this confidence level in order to calculate the combined uncertainty. From (14) with algebraic manipulation,

$$x_2 = \frac{z_2(x_1 - \mu)}{z_1} + \mu. \tag{15}$$

Thus the equivalent voltage limit for the 95% confidence interval is 247.5V, or $p_2 = 7.6\%$. We can now say that $99/10 \equiv 95/7.6$.

Assuming that they are uncorrelated, the two measurement errors (the instrument error, and the voltage error) can be multiplied according to the principle in (4), so that the new measurement error becomes:

$$p_m = \sqrt{0.076^2 + 0.005^2} = 7.603\%. \tag{16}$$

Substituting this new figure into (12) for $\mathrm{CV}_s = 0.5$, we find that the new $\mathrm{CV}_{combined} = 1.003$, 0.3% higher because of measurement uncertainty. This increases the required sample size from 68 to 69 for a 90/10 reporting level. One could argue that the relative contribution of measurement uncertainty is small because the sampling uncertainty dominates. However, the required

sample size also decreases as CV_s decreases. For a stable system with $CV_s = 0.1$, the addition the measurement uncertainty increases the required 90/10 sample size from 3 to 4. Although this seems like a small change, not considering measurement uncertainty and installing too few meters will mean that the M&V report cannot conform to the reporting accuracy requirements.

Measurement Uncertainty for Other Instruments

Parameters other than energy are often measured in order to estimate covariate relationships and calculate energy use. For example, if a relationship between outside air temperature and building energy use can be established, energy savings may be calculated given temperature data from the baseline and reporting periods. Other measurements may include the temperature in ducts or pipes, ambient temperature, humidity, flow rate, wind speed, solar radiation, or machine run time. Unlike electricity which is very regular and precise, the spatial and temporal variability in these cases is significant. For example, the flow rate and temperature in a duct varies between the edge and the centre, and features such as elbows impact flow and heat transfer characteristics for a non-negligible portion of the duct. This is compounded by measurement instrument imprecision much higher than those of electricity meters, as well as a greater sensitivity to operator skill. Because of these complex interactions it is useful to work with general error estimates such as those found in Appendix A5.6 of the ASHRAE Guideline 14-2002 (ASHRAE 2002). Accuracies for these devices are usually between 2% and 5%, but these are only approximations.

For M&V measurements where the parameters measured above are related back to electricity usage using a mathematical model, the impact of measurement uncertainty on overall reporting uncertainty may be higher than figures mentioned above. Consider the case of modelling the impact of boiler insulation. The standing loss of a boiler is influenced by convective and radiative heat transfer. Radiation heat transfer is a function of $T^4_{boiler} - T^4_{surroundings}$. Assume an optimistic value of 2% error with a 95% confidence level on the temperature readings of the outside surface from a thermocouple or infrared thermometer (ASHRAE 2002). In this case, the radiation heat transfer may vary by as much as 23.6% for T_{boiler} = 80°C and $T_{surroundings}$ = 25°C (353.15K and 298.15K). (Usually convective heat transfer dominates and so the overall uncertainty may be lower than this value, but this should be determined on a case-by-case basis.)

Another common example of such complex interactions is the leaking of pipes in underground mines. Technical Note #44 of ASHRAE 14-2002 A5.6.2.5 gives the instrument system error for the bucket/stopwatch technique as 5% (ASHRAE 2002). If all leaks are measured, the overall uncertainty on water waste will be 5% (one can assume a 95% confidence interval, thus 95/5). However, since flow rate output is related non-linearly to input energy for centrifugal pumps, the energy use due to leakage may vary non-linearly, and will depend on the pump and system characteristics. A 5% leak rate uncertainty will then translate in to a larger energy use uncertainty.

These calculations are scenario-specific, but illustrate how modelling uncertainty can be sensitive to measurement uncertainty. The measurement accuracy by itself may be negligible, but the way in which these data are manipulated during calculation could increase reporting uncertainty to unacceptable levels. No general rule will capture the physical complexity of such measurement systems, but by applying the methods outlined in this paper, such uncertainty can be quantified adequately.

Meter Selection and Risk Mitigation

Consider the following Energy Performance Contract example. The project developer has to decide between the two meters described earlier, and he decides to to a cost/benefit analysis taking metering uncertainty into account:

A guaranteed savings model is agreed upon. The project developer guarantees savings G, such that the project developer compensates the owner for any savings shortfall. The profit from any surplus savings are shared between the owner and project developer in the ratio $\beta : (1-\beta)$. Let the energy rate be r \$/kWh. The owner's profit (above guaranteed savings) can be represented as:

$$\text{Owner Profit} = \begin{cases} 0 & \forall\ x \leq G \\ r(\beta x - \beta G) & \forall\ x > G \end{cases} \tag{17}$$

The project developer's profit may be defined as:

$$\text{PD Profit} = \begin{cases} -r(G - x) & \forall\ x \leq G \\ r\left[(1-\beta)x - G(1-\beta)\right] & \forall\ x > G \end{cases} \tag{18}$$

If the probability of realising savings x is $p(x)$, the Expected Value (EV) for the owner is simply the probability of a certain savings scenario, multiplied by its associated cost:

$$EV_{owner} = r\int_{G}^{\infty} (\beta x - \beta G)p(x)dx, \tag{19}$$

and for the project developer:

$$EV_{PD} = -r\int_{-\infty}^{G} (G - x)p(x)dx + r\int_{G}^{\infty} \left[(1-\beta)x - G(1-\beta)\right] p(x)dx. \tag{20}$$

In the context of meter selection, one can reason about this formulation in two ways. First, a more accurate meter may increase the expected value of the project. Or second, a more accurate meter allows the project developer to guarantee more savings (a higher breakeven point for expected value), which makes the tender more competitive. The first option is considered below.

Suppose the project developer estimates true savings to be distributed as N(1000, 300) MWh. The project developer and owner agree that any additional savings will be split 90:10. Thus $\beta = 0.9$. The energy rate is r =R1/kWh (1 South African Rand). The project developer thinks that he is willing to guarantee 700 MWh of savings, to minimise downside risk. The Project developer now wants to decide how to meter the installation, and if it would be worthwhile to install a Class 0.2S meter instead of a Class 0.5S meter. Together with their CT's, the measurement system accuracies are assumed to be 1% and 2% respectively.

Although this can be solve analytically, a standard Monte Carlo simulation can also be used model these two choices. This approach is adopted because of its scalability, should additional complexity be added later. Let $p_m \backsim \mathrm{N}(0, p_m)$, and True Savings $\backsim$ N(1000, 300). Then

$$x = \frac{\text{True Savings}}{1 + p_m}. \tag{21}$$

The project developer finds that for a Class 0.5S meter, his expected value is R6,551, and for the Class 0.2S meter, it is R6,526. Similarly, the owner's expected value is constant at approximately R290,000. This seems to be a counter-intuitive result for two reasons: the numbers are very

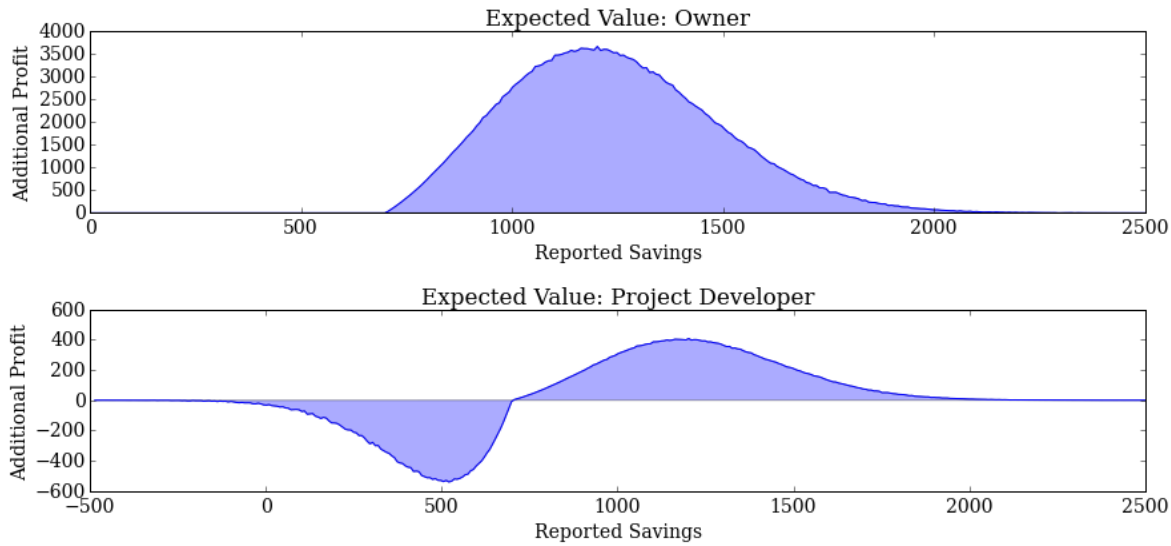

Figure 3: Expected values for project developers and owner under different savings scenarios, given the assumptions in the example problem.

close to one another, and the better meter has a lower expected value. This may be explained by noting that the values for a Monte Carlo simulation are not exact. However, for this specific case it also illustrates that a more accurate meter not only minimises downside risk, but also potential profitability on the upside since the tail of the distribution becomes thinner. This can be visualised from Figure 3. The numbers are, however, so small as to be insignificant. Should a shared savings model be adopted rather than a guaranteed savings model, the downside risk for the project developer would not cancel out the potential for upside profit as it does at the moment. However, more accurate meters may suffer from the same effect in this case. The "tightening" the distribution around a mean value decreases the potentially profitable tail values. It should be noted that this tightening, although less profitable, approaches a truer representation of reality. The "additional" profitability that the project developer and owner lose is solely due to metering error, and should not be viewed as real savings.

Conclusion

In most M&V electricity meter sampling cases where sampling CV is not very low (i.e. stable, predictable processes), the cost of higher accuracy metering (e.g. Class 0.2S over Class 0.5S) is not justified by the increased reporting accuracy. Both "low" and "high" accuracy meters make a negligible uncertainty contribution compared to the uncertainty arising from sampling only a statistically significant fraction of the population. In such cases, a larger sample of the lower accuracy electricity meters will be of greater benefit in minimising overall uncertainty.

In the project planning, metering cost is often a perceived as a barrier to feasibility for the sponsor. Potential projects are not implemented because adequate metering is thought to be too expensive, or because its contribution to reporting uncertainty is thought to be too large. By using the methods outlined in this paper, this contribution can be determined in a mathematically rigorous way, increasing the number of projects eligible for implementation.

For projects measuring independent variables with a strong correlation to energy use, the contribution of these instruments' uncertainty should be calculated, as it may make a material difference to the overall uncertainty. In such cases, the combination of a low cost electricity meter with a high-accuracy covariate measurement instrument should be considered. For example, where energy use is sensitive to outside air temperature, an accurate and professionally installed temperature measurement and logging device, combined with a 0.5S energy meter, could yield more accurate results than a more expensive 0.2S energy meter and weather bureau data.

For energy performance contracting projects using both the guaranteed and shared savings models, it is found that more accurate meters do not necessarily increase the project's expected value for the project developer or the owner. A more accurate meter reduces both upside and downside risk. For such cases, contract requirements rather than profit should drive metering decisions.

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